Knight (2002) *Nature*

The University of Manchester

An Integrated Active Learning Environment for Advanced Engineering Mathematics

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Department of Mathematics, University of Manchester

FSE TA Mini-Conference: “Emerging Stronger”
28th May 2021

Acknowledgements

Support

Maths T&L Office

UoM e-Learning Team

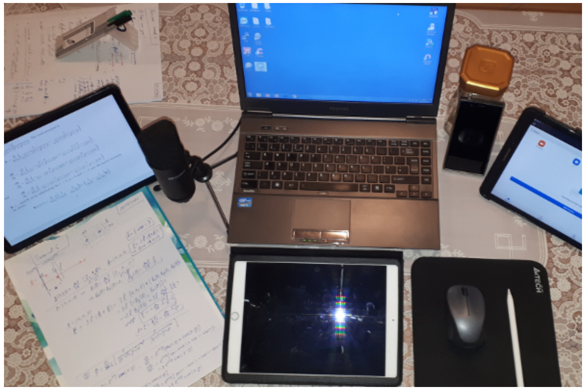
Infrastructure

Chris Sangwin and STACK Users Group (Edinburgh)

Advanced Engineering Mathematics MATH29681

- Year 2 course unit of three parts
  (Integral Transforms, Vector Calculus & Linear Algebra)
- Two unit leaders
- 250+ students from Electrical and Electronic Engineering
- Highly diverse background and level of training
  (including direct-entry overseas students)

T&L Environment



Advanced Engineering Mathematics MATH29681

2019/20 Structure

ILOs

Lectures + Tutorial (weekly)

Practice Test 1

Test 1

Revision

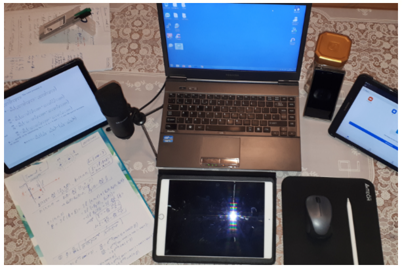
Practice Test 2

Test 2

Revision

Exam

Advanced Engineering Mathematics MATH29681



2020/21

ILOs

Blackboard Quizzes and Milestones

Piazza Learning Environment

Review + Tutorial sessions (weekly)

Revision Session

STACK Practice 1

Test 1

STACK Practice 2

Test 2

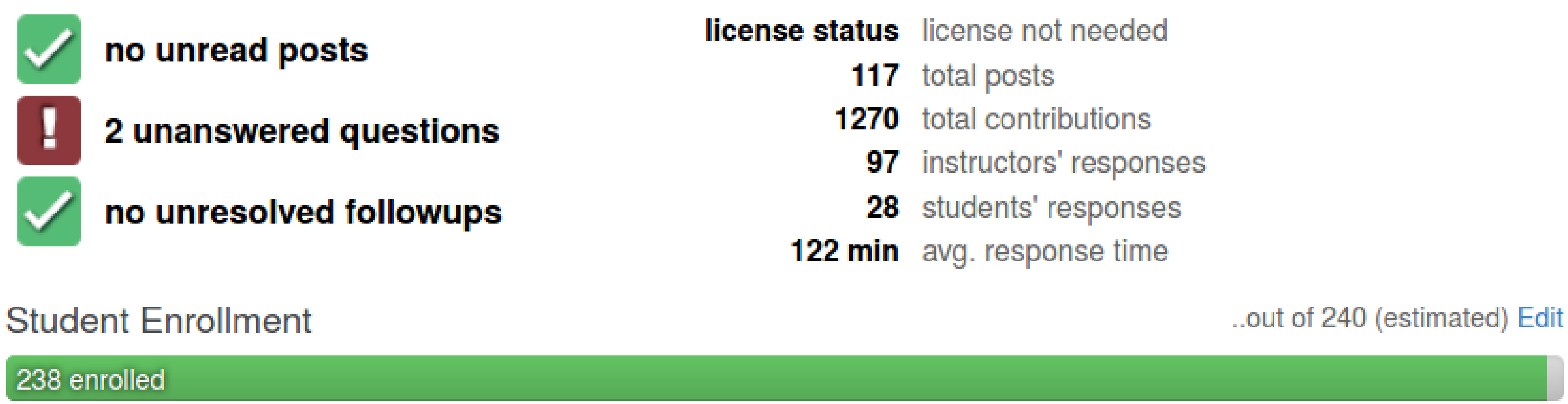
STACK Exam

STACK Revision

Piazza

Piazza

- Integrated in Blackboard
- Peer-to-peer and lecturer-led support
- Anonymous re-posting – replaces emails
- Live and persistent polls
- LaTeX support, etc.



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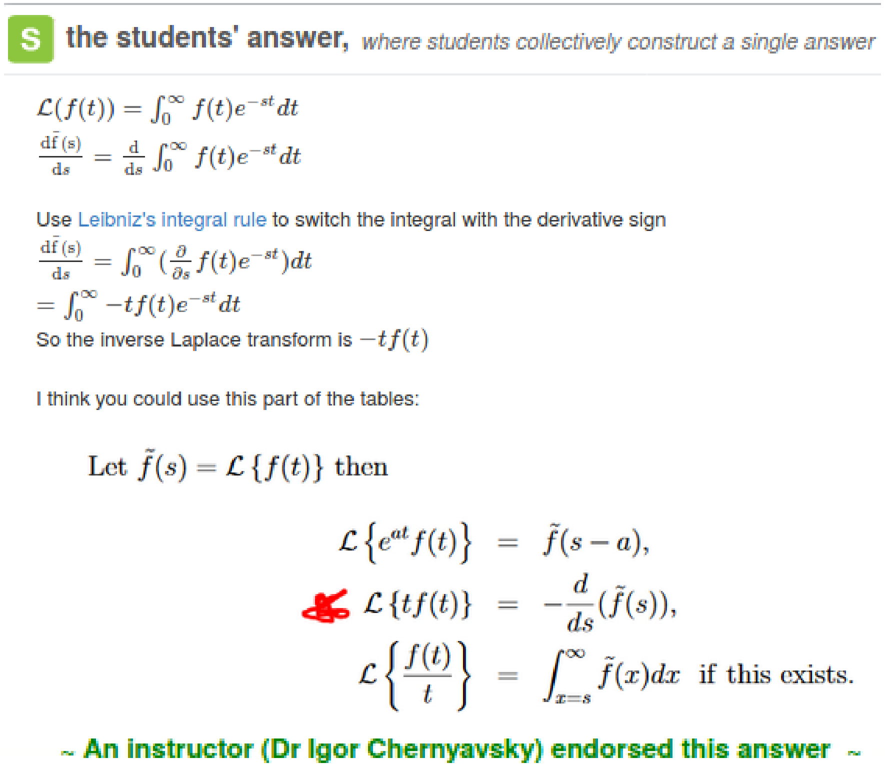


the students' answer, *where students collectively construct a single answer*

$\mathcal{L}(f(t)) = \int_0^\infty f(t)e^{-st}dt$

$\frac{\mathrm{d}\tilde{f}(s)}{\mathrm{d}s} = \frac{\mathrm{d}}{\mathrm{d}s}\int_0^\infty f(t)e^{-st}dt$

Use Leibniz's integral rule to switch the integral with the derivative sign

$\frac{\mathrm{d}\tilde{f}(s)}{\mathrm{d}s} = \int_0^\infty (\frac{\partial}{\partial s} f(t)e^{-st})dt$

$= \int_0^\infty -tf(t)e^{-st}dt$

So the inverse Laplace transform is $-tf(t)$

I think you could use this part of the tables:

Let $\tilde{f}(s) = \mathcal{L}\{f(t)\}$ then

$$\mathcal{L}\left\{e^{at}f(t)\right\} = \tilde{f}(s-a),$$
$$\mathcal{L}\{tf(t)\} = -\frac{d}{ds}(\tilde{f}(s)),$$
$$\mathcal{L}\left\{\frac{f(t)}{t}\right\} = \int_{x=s}^\infty \tilde{f}(x)dx \text{ if this exists.}$$

~ An instructor (Dr Igor Chernyavsky) endorsed this answer ~

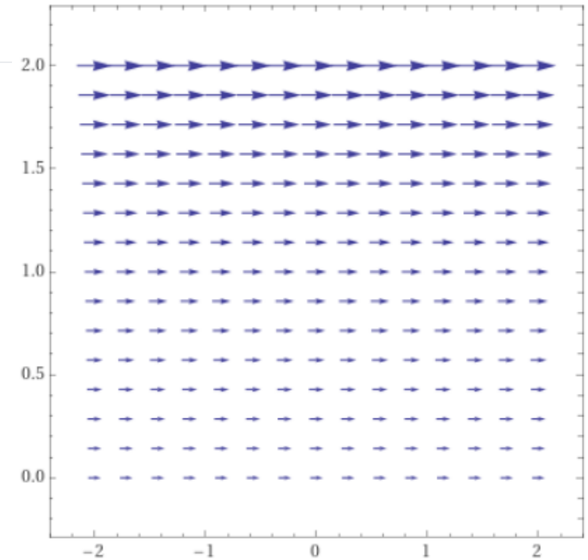
Piazza: Polling

poll @78

Week 7: Groupwork task 1

Is the vector field $\mathbf{F}(x, y) = \left(\frac{1}{(y-5)^2},\ 0 \right)$ depicted below a conservative vector field?

If it is, find the corresponding scalar potential Φ such that $\mathbf{F} = \nabla\Phi$.



- Yes, $\Phi = x^2 + (y-5)^2$
- Yes, $\Phi = x^2 + (y-5)^2 + C$
- Yes, $\Phi = \frac{1}{x^2+(y-5)^2}$
- Yes, $\Phi = \frac{1}{\sqrt{x^2+(y-5)^2}}$
- Yes, $\Phi = \frac{x}{(y-5)^2}$
- No.
- Not enough information to conclude.

2 (8% of users)	Yes, $\Phi = x^2 + (y-5)^2$
6 (23% of users)	Yes, $\Phi = x^2 + (y-5)^2 + C$
1 (4% of users)	Yes, $\Phi = \frac{1}{x^2+(y-5)^2}$
2 (8% of users)	Yes, $\Phi = \frac{1}{\sqrt{x^2+(y-5)^2}}$
4 (15% of users)	Yes, $\Phi = \frac{x}{(y-5)^2}$
9 (35% of users)	No.
2 (8% of users)	Not enough information to conclude.

Explanation: Correct answer: No.

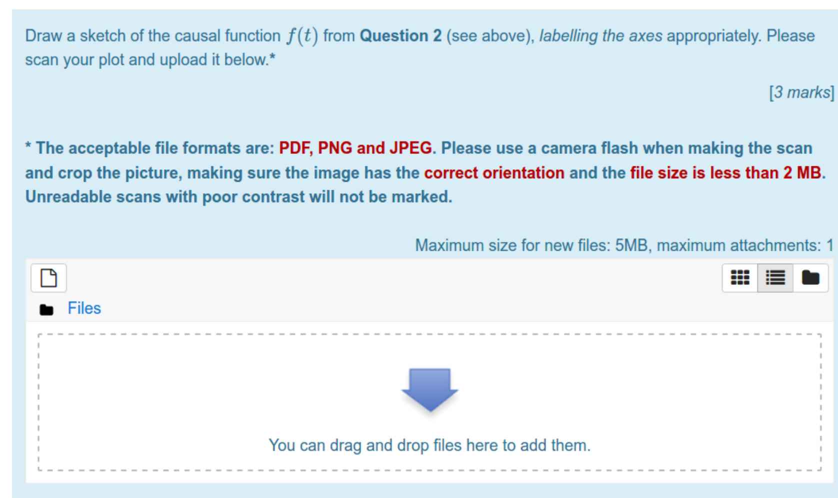
STACK

STACK + Moodle

- Robust randomisation
- Carry-through calculation errors and consistent marking
- Instant (or deferred) and detailed feedback

STACK + Moodle

- Robust randomisation
- Carry-through calculation errors and consistent marking
- Instant (or deferred) and detailed feedback
- Could be mixed with semi-automatic marking:
  - short textual justification
  - uploading a graph sketch



Draw a sketch of the causal function $f(t)$ from **Question 2** (see above), *labelling the axes* appropriately. Please scan your plot and upload it below.*

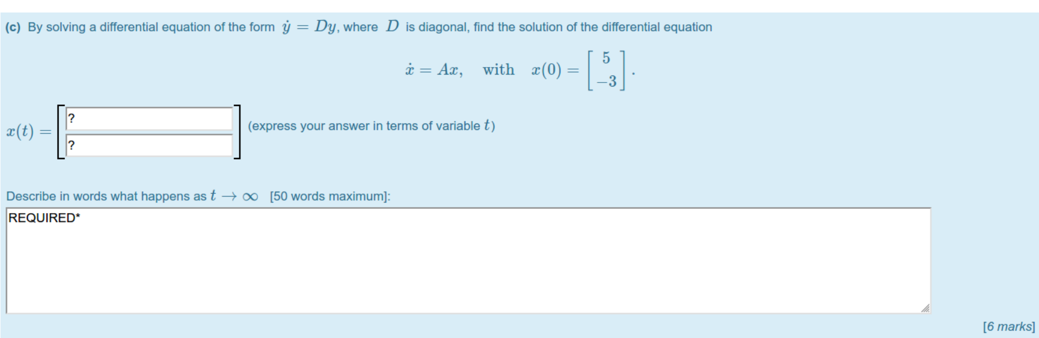
[*3 marks*]

*** The acceptable file formats are: PDF, PNG and JPEG. Please use a camera flash when making the scan and crop the picture, making sure the image has the correct orientation and the file size is less than 2 MB. Unreadable scans with poor contrast will not be marked.**

Maximum size for new files: 5MB, maximum attachments: 1

Files

You can drag and drop files here to add them.



(c) By solving a differential equation of the form $\dot{y} = Dy$, where D is diagonal, find the solution of the differential equation

$$\dot{x} = Ax, \quad \text{with} \quad x(0) = \begin{bmatrix} 5 \\ -3 \end{bmatrix}.$$

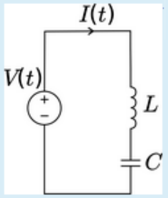
$x(t) = \begin{bmatrix} ? \\ ? \end{bmatrix}$ (express your answer in terms of variable t)

Describe in words what happens as $t \to \infty$ [50 words maximum]:

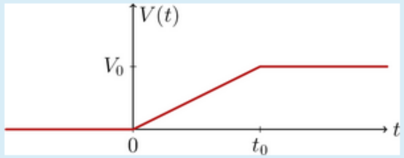
REQUIRED*

[*6 marks*]

STACK: Adaptive Multi-part Questions



An electrical LC circuit (see the diagram) consists of a serially connected capacitor with capacitance
$C = \frac{2}{9}$
and an inductor with inductance
$L = \frac{1}{2}$
(both given in normalised dimensionless units).



At time $t = 0$ there is no charge at the capacitor and no electric current in the circuit. An external voltage $V(t)$ (in dimensionless units) is applied to the circuit, increasing linearly from $V(t=0) = 0$ to $V(t = t_0) = V_0$, and is then kept constant after that time ($V = V_0$ for $t \geq t_0$; see the plot).

Here $t_0 = 4$ and $V_0 = 108$ (in dimensionless units).

(i) Express the externally applied voltage $V(t)$ as a function of time for $t > 0$.
Use **u(t)** to denote the unit step function in your answer:
$V(t) =$ REQUIRED* (express your answer as a function of t)

[*3 marks*]

(ii) Assuming the electric current $I(t)$ in the circuit described above obeys the equation

$$\frac{1}{2}\frac{\mathrm{d}\,I(t)}{\mathrm{d}\,t} + \frac{9}{2}\int_0^t I(\tau)\,d\tau = V(t)\,,$$

use the Laplace transform to find the associated *Transfer Function* $G(s)$ that links the input to the output of the system:
$G(s) =$ REQUIRED* (express your answer as a function of s)

Find the corresponding *Impulse Response Function* $i(t)$ that characterises the electric current in response to a unit impulse voltage input $V(t) = \delta(t)$:
$i(t) =$ REQUIRED* (express your answer as a function of t)

[*5 marks*]

(iii) For the external voltage $V(t)$ found in part **(i)**, apply the Laplace Transform to the equation given in **(ii)** and express the transformed current $\bar{I}(s)$ in the s-domain:
$\bar{I}(s) =$ REQUIRED* (give your answer as a function of s)

[*6 marks*]

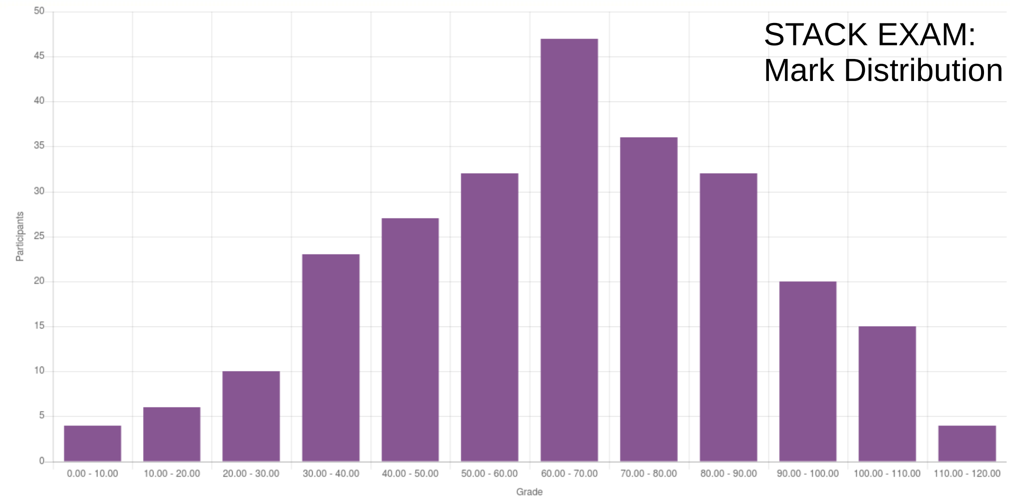
(iv) Use the results of **(iii)** and the inverse Laplace transform to solve the ordinary differential equation for the LC circuit and find the value of the electric current $I(t_0)$ at time $t_0 = 4$:
$I(4) =$ REQUIRED*

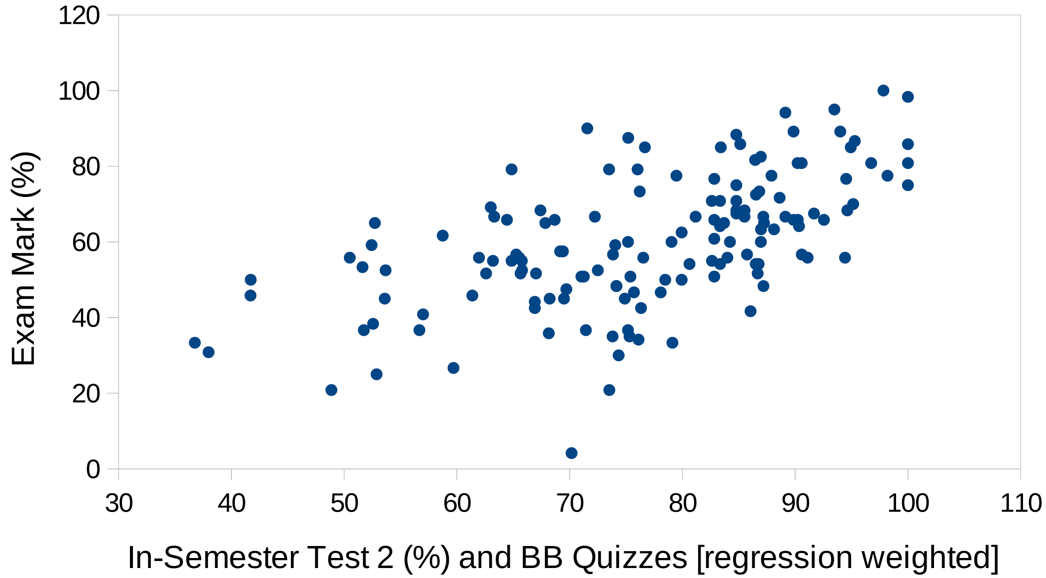
[*6 marks*]

Learning Outcomes

Statistics

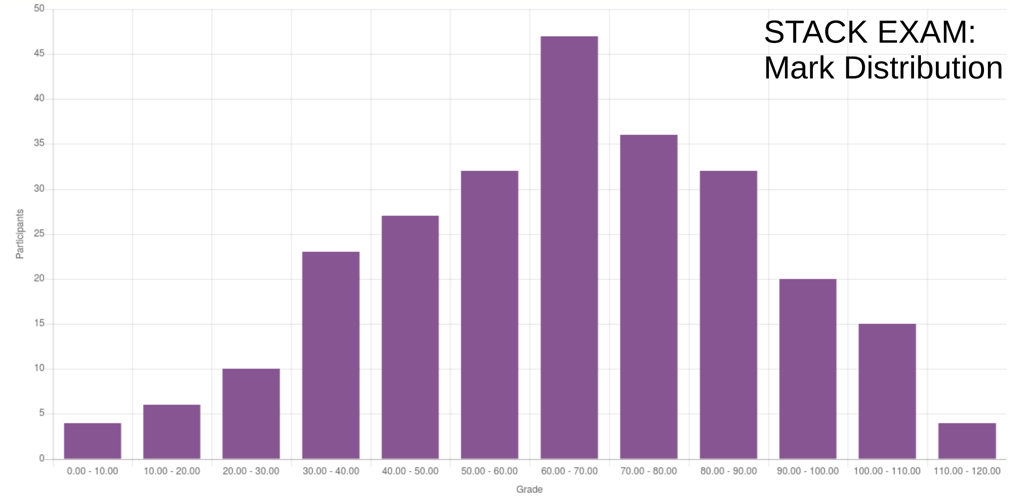
- Robust semi-automatic marking
- Quantitative analysis of learning outcomes

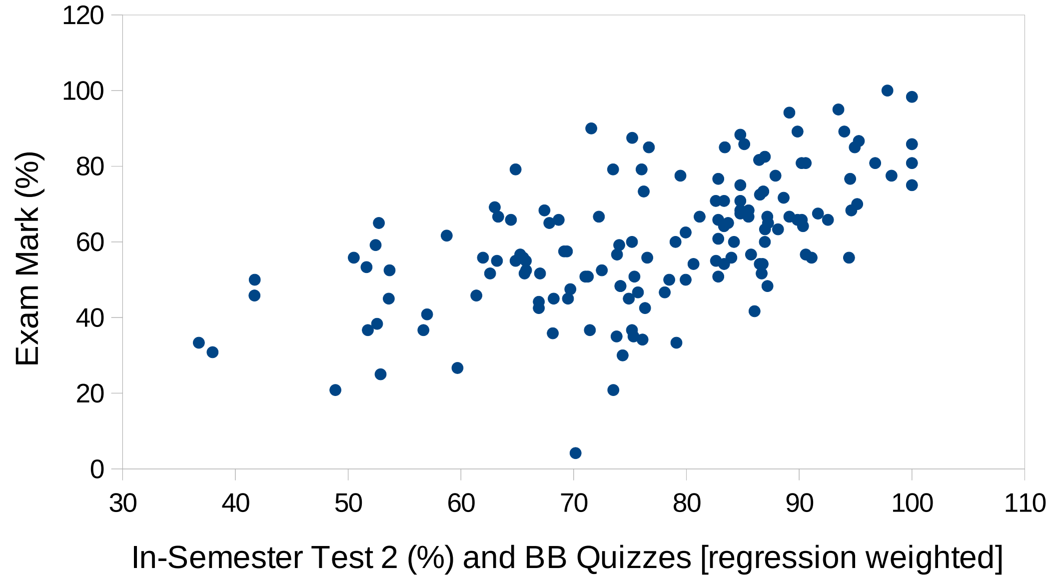




Statistics

- Robust semi-automatic marking
- Quantitative analysis of learning outcomes
    - **2019/20**: early engagement is a predictor of success
    - **2020/21**: sustained engagement is a predictor of success





Feedback

- “The Milestones were also well integrated with Piazza and gave a quick overview of what you've learned by taking the course.”
- “Piazza forum is a nice idea so answers to questions can be seen by everyone.”
- “The practice exam was a good opportunity to check for any gaps in knowledge before attempting the assessed version.”
- “I found the feedback solutions very detailed and helpful.”
- “[...] to master the skills of accurate output of our mathematical knowledge, to apply them in real-life problems.”
- “The additional content was cool too, although it was hard for me to give it any real attention with the need to prioritise core content.”
- “I would say the amount of work was quite hefty.”

Summary & Outlook

- The e-learning environment for Maths is a continuum
- Active learning can be quantified
- The extra effort pays off for large classes.

Thank you for listening!

- STACK: stack-assessment.org
- Piazza: piazza.com
- FSE e-Learning Portal: elearning.fse.manchester.ac.uk